Wave Packet Dynamics and Chaotic Eigenstates

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Summary. We consider phase-space correlations in the Husimi densities of eigenstates of quantum chaotic systems. It is explained that the existence of such correlations follows from the validity of short-time Gaussian wave packet dynamics. Using this semiclassical approximation and some results from random-matrix theory we derive an expression for the correlation coefficient of the Husimi densities at two different points in phase space which are connected by a short classical trajectory. An explicit expression for this correlator is given for the case of a single iteration of the quantum kicked rotator.

1 Introduction

Wave packet propagation is the most straightforward approach to a semiclassical description of quantum dynamics. The main idea is due to Heller [1]. One approximates a quantum state by a wave packet with a predefined shape that depends on a few classical parameters only. For example this could be a Gaussian in phase space with given mean values and widths for position and momentum. One can then derive classical equations of motion for these parameters although they do account at least partially for quantum effects like the uncertainty principle. Of course the choice of the wave packets is somewhat arbitrary, and it may require a fair amount of physical insight to identify for a given problem the relevant parameters and to find the optimal compromise between accuracy and numerical tractability. The latter is an important issue when large systems of interacting particles are of interest, and this is probably the most important field for applications of wave packet dynamics. An example is the problem of a partially ionized hydrogen plasma which was studied by Werner Ebeling and his students some 10 years ago [2,3]. Fortunately I was also part of his group around that time, and thus Werner Ebeling introduced me to the subject of wave packet dynamics in a number of discussions. Unfortunately, however, it took almost ten years before I finally managed to apply this lesson to my own field of research, namely quantum chaos. It turns out that simple Gaussian wave packet dynamics can provide valuable information about long-distance correlations in the stationary states of quantum chaotic systems [4]. In the present contribution we will further develop this novel application of wave packet dynamics.

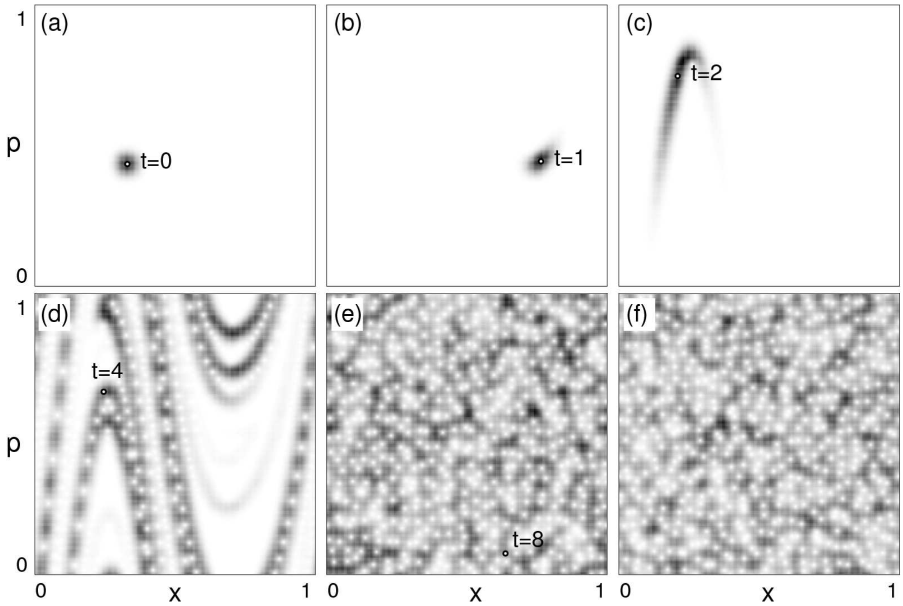

Fig. 1. Time evolution of a quantum state for the modified kicked rotator with $k = 7.5$ and $N = 512$ in Husimi representation. (a) An initial coherent state at $\xi_0 = (x_0, p_0) = (0.33, 0.435)$ is prepared. The phase space points ξ_t marked in (b)-(e) are the classical iterates of this point. (b) At $t = 1$ the state is still approximately Gaussian and centered at ξ_1. (c) $t = 2$: The state is exponentially stretched along the unstable manifold of the trajectory and begins to deviate strongly from a Gaussian shape (ellipse). (d) $t = 4$: The state is folded back into the unit cell several times and begins to develop a complicated structure. (e) $t = 8$: The quantum state covers the whole available phase space. (f) For comparison a random state is shown in Husimi representation.

In quantum chaos [5] one deals with quantum systems in the semiclassical limit where the number of relevant basis states $N \sim \hbar^{-d}$ (the volume ratio between the classically accessible part of phase space and a Planck cell) diverges and still quantum effects remain important. One would like to understand in detail how the classical and the quantum dynamics are related in this regime, in particular in a situation where the classical dynamics is chaotic and thus intrinsically unstable. Gaussian wave packet dynamics becomes exact in the semiclassical limit $N \to \infty$ if one considers the dynamics over a given finite time t. On the other hand, for any finite value of N this trivial quantum-classical correspondence breaks down after a relatively short time $t^* \sim \lambda^{-1} \ln N$ if the classical dynamics is chaotic with Lyapunov exponent λ. In the semiclassical limit $N \to \infty$ this time scale is much shorter than the Heisenberg time $t_H \sim N$, which marks the transition to a quasiperiodically recurrent dynamics that is due to the discreteness of the quantum spectrum.

Fig. 1 illustrates this behavior for the so-called quantum kicked rotator that will be introduced in Section 3 and serves as our model for Hamiltonian chaos. Repeated application of the quantum propagator generates a sequence of states $|\psi_t\rangle$ whose phase-space densities are shown in Figs. 1a-e. In this Husimi representation any Gaussian wavepacket results in an elliptic density distribution, and coherent states like the initial wavepacket in Fig. 1a appear circular. Gaussian wavepacket dynamics assumes that all time-evolved states are Gaussian, but the figure shows that this is qualitatively wrong for $t \gg 1$ despite the small effective value of Planck's constant. Due to classical chaos, the wave packet is first stretched exponentially along its unstable direction and then folded back into the unit cell several times. This gives rise to a complicated structure which cannot anymore be described by just a few parameters. After some additional iterations of the quantum propagator the state covers the whole available phase space and looks qualitatively similar to the random state[1] shown in Fig. 1f. We conclude that wave packet dynamics provides only a very limited insight into the quantum dynamics of chaotic systems.

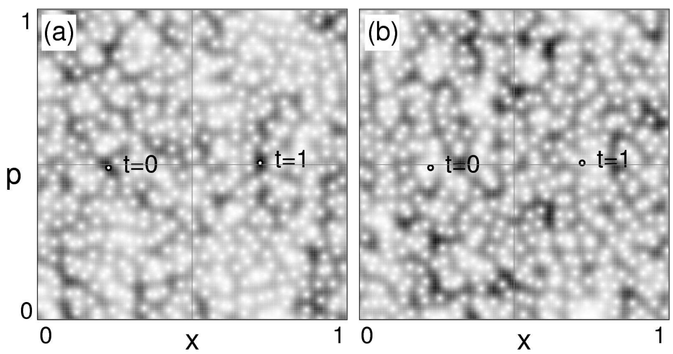

Fig. 2. Husimi projection of two selected eigenstates of the modified kicked rotator. The point (x_1, p_1) denoted by $t = 1$ is the classical iterate of the point (x_0, p_0). Both points are close to a maximum (a) or minimum (b) of the eigenstate, respectively.

[1] For this state, the N amplitudes in position representation were chosen as complex random numbers. Subsequently the state was normalized.

Of course the complete information about the quantum dynamics is contained in the eigenvalues and eigenfunctions of the Hamiltonian. Fig. 2 shows two typical examples for the kicked rotator. On first sight these states appear to be very similar to the random state of Fig. 1f. In fact this observation agrees with the prediction of random-matrix theory (RMT) which has been the main theoretical tool for the statistical analysis of quantum systems with chaotic classical analogue for more than 20 years [5]. Within RMT one replaces the Hamiltonian by a random matrix drawn from a suitable ensemble and can then calculate many physically relevant quantities from an average over this ensemble. Despite the great success of this approach one may loose important physics if all system-specific properties are neglected in this way. For example, although the eigenstates of Fig. 2 have nothing in common with localized wave packets, the validity of wave packet dynamics for short times represents a system-specific constraint on these states. It results in a correlation between the intensities of an eigenstate at different points in phase space which would not be expected within RMT. Qualitatively we can understand this effect as follows: Suppose that by chance an eigenstate has a high intensity at some point $\xi_0 = (x_0, p_0)$. For example, this is the case in Fig. 2a for the spot denoted by $t = 0$. We can think of an additional localized wave packet superimposed on the fluctuating background of the eigenstate. From wave packet dynamics we expect that in the course of time this additional intensity is transported to a different location in phase space, namely ξ_1 $(t = 1)$ in Fig. 2a. On the other hand, up to an irrelevant phase, the eigenstate is invariant under time evolution. Therefore the state must have a high intensity also at ξ_1 and this is confirmed in Fig. 2a. ¿From a similar argument we can conclude that low intensities at ξ_0 and ξ_1 tend to coincide (Fig. 2b) and, more generally, that the intensities along a short classical trajectory are correlated.

A quantitative theory for these correlation was outlined and numerically confirmed in [4]. Here it is our purpose to give a detailed derivation of the results presented there. We will first reduce the calculation of the Husimi correlator along a classical trajectory to the propagation of a Gaussian wave packet (Section 2). Although this step does not rely on a specific model system, chaotic classical dynamics is implicitly assumed as we make use of some results from random-matrix theory. Then we derive in Section 3 explicit expressions for the propagation of Gaussian wave packets in kicked Hamiltonian systems and simplify them in the special case for which numerical results were presented in [4] (propagation of a coherent initial state over a single time period of the kicked rotator). Finally we end with some concluding remarks in Section 4.

2 The Husimi correlator

In this section we give a quantitative derivation of the correlations in the Husimi densities of eigenstates at two different points ξ_0 and ξ_t in phase space which are connected by a short classical trajectory of length t. We denote the (effective) dimension of the Hilbert space by N and consider the N eigenstates of the quantum propagator over time t,

$$\hat{U}^t|n\rangle = e^{i\lambda_n t}|n\rangle \, . \tag{1}$$

These states form a complete orthonormal basis, $\hat{I} = \sum_{n=1}^{N} |n\rangle\langle n|$. An average over the eigenstates will be denoted by $\langle \cdot \rangle_n$.

In order to map quantum states into the classical phase space we introduce coherent states $|\xi_0\rangle$ by their position representation

$$\langle x|\xi_0\rangle = (\hbar\pi)^{-1/4} \, e^{-(x-x_0)^2/2\hbar + ip_0(x-x_0)/\hbar} \, . \tag{2}$$

These are minimum uncertainty wave packets centered at $\xi_0 = (x_0, p_0)$ with equal widths in position and momentum. They are normalized but not orthogonal and provide an overcomplete basis. The Husimi density of an arbitrary quantum state $|\psi\rangle$ at a point ξ in phase space is defined as

$$H_\psi(\xi) = |\langle\xi|\psi\rangle|^2 \, . \tag{3}$$

We consider the Husimi densities of the eigenstates $|n\rangle$ (Fig. 2). The mean value of this density is

$$\langle H_n(\xi)\rangle_n = N^{-1} \sum_{n=1}^{N} \langle\xi|n\rangle\langle n|\xi\rangle = N^{-1} \, , \tag{4}$$

if we fix the point ξ and average over n. The fluctuations around the mean,

$$\delta H_n(\xi) = H_n(\xi) - N^{-1} \, , \tag{5}$$

will be the object of our interest. As a measure for the correlation of the Husimi densities at two different phase-space points ξ and ξ' we consider the correlation coefficient (normalized covariance)

$$C_H(\xi';\xi) = \frac{\langle \delta H_n(\xi) \, \delta H_n(\xi')\rangle_n}{\sqrt{\langle \delta^2 H_n(\xi)\rangle_n \langle \delta^2 H_n(\xi')\rangle_n}} \, . \tag{6}$$

Such correlations exist trivially if ξ and ξ' are very close in phase space since then the corresponding coherent states have a big overlap. Moreover, according to the arguments given in the introduction, correlations are expected along short classical trajectories between the density $H_{0,n} = H_n(\xi_0)$ at a point ξ_0 and the density $H_{t,n} = H_n(\xi_t)$ at its iterate ξ_t. In order to calculate this correlation we consider the time evolution of $|\xi_0\rangle$. For short times $t \lesssim t^*$ this yields a wave packet which is localized in the vicinity of ξ_t (Fig. 1). We decompose this wave packet into two contributions,

$$\hat{U}^t|\xi_0\rangle = a_t|\xi_t\rangle + r_t|\rho_t\rangle\,, \tag{7}$$

namely the coherent state $|\xi_t\rangle$ and a remainder $|\rho_t\rangle$. This is a normalized state which is not explicitly specified but orthogonal to $|\xi_t\rangle$, $\langle\rho_t|\xi_t\rangle = 0$. Therefore the expansion coefficients satisty

$$|a_t|^2 + |r_t|^2 = 1\,. \tag{8}$$

Due to the orthogonality of the two states we can also assume that the projections

$$\xi_{t,n} = \langle n|\xi_t\rangle \tag{9}$$
$$\rho_{t,n} = \langle n|\rho_t\rangle \tag{10}$$

of an eigenstate $|n\rangle$ on $|\xi_t\rangle$ and $|\rho\rangle$ have independent statistics,

$$P(\rho_{t,n}, \xi_{t,n}) = P(\rho_{t,n})\,P(\xi_{t,n})\,. \tag{11}$$

This random-matrix type assumption is expected to hold in the semiclassical limit unless ξ_t is close to a short periodic orbit or a symmetry line, see [4] for a brief discussion of such exceptional points. We can now multiply the identity

$$\begin{aligned} H_{0,n} &= |\langle n|\xi_0\rangle|^2 && (12)\\ &= |\langle n|\hat{U}^t|\xi_0\rangle|^2 && (13)\\ &= |\langle n|a_t\xi_t\rangle + \langle n|r_t\rho_t\rangle|^2 && (14)\\ &= |a_t|^2\,|\xi_{t,n}|^2 + 2\Re\, a_t^* r_t \xi_{t,n}^* \rho_{t,n} + |r_t|^2|\rho_{t,n}|^2 && (15) \end{aligned}$$

with $H_{t,n} = |\xi_{t,n}|^2$ and average over n. Then the first term can be expressed in terms of the *inverse participation number* (IPR) in Husimi representation and gives $|a_t|^2\,N^{-1}\mathcal{I}_t$. Here, the IPR is defined as

$$\mathcal{I}_t = \sum_n H_{t,n}^2 = \sum_n |\langle n|\xi_t\rangle|^4 \tag{16}$$

and has the following basic property: We have $\mathcal{I}_t = 1$ if the coherent state $|\xi_t\rangle$ is an eigenstate of the propagator and $\mathcal{I}_t = N^{-1} \ll 1$ if this state is euqally distributed over all eigenstates. In general $N^{-1} \le \mathcal{I}_t \le 1$ can be considered as a measure for the localization of $|\xi_t\rangle$ in the eigenbasis $|n\rangle$.

The second term of Eq. (15) vanishes upon averaging since, according to Eq. (11), the phases from $\xi_{t,n}^*$ and $\rho_{t,n}$ are uncorrelated. Also the third term factorizes and gives

$$\langle |\xi_{t,n}|^2\,|r_t|^2|\rho_{t,n}|^2\rangle_n = \langle |\xi_{t,n}|^2\rangle_n\,|r_t|^2\langle|\rho_{t,n}|^2\rangle_n \tag{17}$$
$$= N^{-2}\,|r_t|^2 \tag{18}$$
$$= N^{-2}\,(1-|a_t|^2) \tag{19}$$

where we have used that the eigenstates $|n\rangle$ form a complete basis and also Eq. (8) for the last line. Hence we find

$$\langle H_{0,n}H_{t,n}\rangle_n = N^{-1}|a_t|^2\mathcal{I}_t + N^{-2}\,(1-|a_t|^2) \tag{20}$$

and further

$$\begin{aligned}\langle \delta H_{0,n}\,\delta H_{t,n}\rangle_n &= \langle H_{0,n}H_{t,n}\rangle_n - N^{-2}\\ &= |a_t|^2 N^{-1}(\mathcal{I}_t - N^{-1})\,.\end{aligned} \tag{21}$$

Moreover we have

$$\begin{aligned}\langle \delta^2 H_{t,n}\rangle_n &= \langle H_{t,n}^2\rangle_n - N^{-2}\\ &= N^{-1}(\mathcal{I}_t - N^{-1})\,.\end{aligned} \tag{22}$$

After substitution of these results we find for the Husimi correlator

$$\begin{aligned}C_H(\xi_0;\xi_t) &= \frac{\langle \delta H_{0,n}\,\delta H_{t,n}\rangle_n}{\sqrt{\langle \delta^2 H_{0,n}\rangle_n \langle \delta^2 H_{t,n}\rangle_n}}\\ &= |a_t|^2\sqrt{\frac{\mathcal{I}_t - N^{-1}}{\mathcal{I}_0 - N^{-1}}}\,.\end{aligned} \tag{23}$$

Further progress relies now on another result from random-matrix theory, according to which we have the estimate $\mathcal{I} = 2N^{-1}$ for an arbitrary state. Deviations from this result have beenn studied in the past by Heller, Kaplan and others [6]. It was found, that enhanced localization (higher values of the IPR) are found in the vicinity of short periodic orbits with relatively low Lyapunov exponent. However, in our case this *scarring* effect is not important for two reasons: (i) We must exclude the vicinity of short periodic orbits anyway since there factorization Eq. (11) cannot be justified and (ii) if ξ_0 is close to a short periodic orbit so is ξ_t. Thus we can assume $\mathcal{I}_0 = \mathcal{I}_t$ and end up with a very simple estimate for the Husimi correlation along short trajectories,

$$C_H(\xi_0;\xi_t) = |a_t|^2 = |\langle \xi_t|\hat{U}|\xi_0\rangle|^2\,. \tag{24}$$

Now the connection between eigenfunction statistics and Gaussian wave packet dynamics is obvious. In order to obtain an explicit expression for the phase-space correlator, one has to approximate the time evolution of a Gaussian wave packet (a coherent state) and to project the result to another Gaussian. We will do this in the following section for a specific model system.

3 The kicked rotator

In this section we consider a particle that is periodically forced by very short pulses such that the Hamiltonian can be written as

$$H(p,x,t) = T(p) + V(x) \sum_{n=-\infty}^{+\infty} \delta(t-n) \tag{25}$$

with $T(p) = p^2/2$ (kinetic energy) and

$$V(x) = \frac{k}{(2\pi)^2} \left[\cos\frac{\alpha\pi}{2} \cos 2\pi x + \frac{1}{2} \sin\frac{\alpha\pi}{2} \sin 4\pi x \right] \tag{26}$$

(potential energy). For historical reasons this model is called modified kicked rotator. Due to the special time dependence in Eq. (25) the classical dynamics reduces to a simple map $\xi_{n+1} = \mathcal{T}\xi_n$ for position and momentum right after the kicks. Explicitly it is given by

$$x_{n+1} = x_n + T'(p_n) \tag{27}$$

$$p_{n+1} = p_n - V'(x_{n+1})\,. \tag{28}$$

In addition to the time-periodic forcing, the phase space is periodic in position and momentum too. We have choosen dimensionless variables p, x, t such that all periods are unity, i.e., we can restrict Eqs. (27), (28) to the unit torus $x+1 \equiv x$ and $p+1 \equiv p$. For $\alpha = 0$ Eqs. (27), (28) are Chirikov's standard map [7] which is one of the most prominent paradigms of Hamiltonian chaos. However, for our purpose it is useful to set $\alpha = 0.1$ in order to break an unwanted symmetry. Moreover we use $k = 7.5$ for all numerical calculations since then the classical dynamics of the model is completely chaotic. Quantum mechanically the system is described by the time evolution operator over one period

$$\hat{U} = e^{-i\,V(\hat{x})/\hbar}\, e^{-i\,T(\hat{p})/\hbar}\,. \tag{29}$$

The convenient factorization into two terms depending on position and momentum, respectively, is again a consequence of the δ-kicks [8]. In Eq. (29) $\hbar$ is a dimensionless constant which accounts for the ratio between the phase-space volume and Plancks constant. We assume $\hbar = 2\pi/N$ in order to ensure that $\hat{U}$ is periodic in x and p. For this choice N is the effective dimension of the Hilbert space, and hence also the dimension of the unitary operator $\hat{U}$.

A general Gaussian state for a one-dimensional model like the kicked rotator has in position representation the form

$$g(x_0, p_0, \mathit{\Delta}_x, \mathit{\Delta}_p, \mathcal{N}, s; x) = \mathcal{N} \exp\left(-A(\mathit{\Delta}_x, \mathit{\Delta}_p, s)(x-x_0)^2 + i\frac{p_0(x-x_0)}{\hbar} \right) \tag{30}$$

with

$$A(\Delta_x, \Delta_p, s) = \frac{1 + \frac{2si}{\hbar}\sqrt{\Delta_x^2 \Delta_p^2 - \left(\frac{\hbar}{2}\right)^2}}{4\Delta_x^2} \,. \tag{31}$$

The parameters x_0, p_0 and Δ_x^2, Δ_p^2 are the expectation values and variances of position and momentum, respectively. For a normalized wave packet the complex prefactor $\mathcal{N} = |\mathcal{N}|e^{i\gamma}$ has modulus $|\mathcal{N}| = (2\pi\Delta_x^2)^{-1/4}$. γ is an overall phase. This phase and the sign $s = \pm 1$ do not influence the physical properties of the wave packet. In momentum representation we have for the same state

$$\tilde{g}(x_0, p_0, \Delta_x, \Delta_p, \mathcal{N}, s; p) = \tilde{N} \exp\left(-\tilde{A}(\Delta_x, \Delta_p, s)(p - p_0)^2 - i\frac{x_0(p - p_0)}{\hbar}\right) \tag{32}$$

where

$$\tilde{A}(\Delta_x, \Delta_p, +s) = A(\Delta_p, \Delta_x, -s) \tag{33}$$

and

$$\tilde{\mathcal{N}} = \frac{\mathcal{N}}{\sqrt{2A\hbar}}\, e^{-ix_0p_0/\hbar} = \frac{e^{i(\gamma - x_0p_0/\hbar - \arg(A)/2)}}{(2\pi\Delta_p^2)^{1/4}} \tag{34}$$

For coherent states the imaginary part in Eq. (31) is zero, and hence the sign s is immaterial.

We wil now approximate the action of the quantum propagator $\hat{U}^t$ on a Gaussian wave packet. For this purpose we assume that the width in both, position and momentum, is $\sim \hbar$ and thus for $\hbar \to 0$ much smaller than any classical scale. Then we can expand $V(x)$ and $T(p)$ around the point (x_0, p_0) where the wave packet is centered. If the expansion is restricted to the quadratic approximation

$$\begin{aligned} T(p) &\approx T_0 + T_0'(p - p_0) + \frac{T_0''}{2}\,(p - p_0)^2 \\ V(x) &\approx V_0 + V_0'(x_0)(x - x_0) + \frac{V_0''}{2}\,(x - x_0)^2 \end{aligned} \tag{35}$$

(with $T_0 \equiv T(x_0)$, $T_0' \equiv T'(x_0)$, $\ldots$) the wave packet remains Gaussian. However, its parameters are modified in the following way. After applying the operator related to the free propagation between the kicks (the second term in Eq. (29)) we have

$$e^{-(i/\hbar)\,T(p)}\tilde{g}(x_0, p_0, \Delta_{x0}, \Delta_{p0}, \mathcal{N}_{01}, s_0; p) \approx \tilde{g}(x_1, p_0, \Delta_{x1}, \Delta_{p0}, \mathcal{N}_{01}, s_{01}; p)\,, \tag{36}$$

i.e., mean and variance of p are unaffected, while the mean position moves in agreement with the classical dynamics Eq. (27) to $x_1 = x_0 + T_0'$ and an additional overall phase is acquired, $\mathcal{N}_{01} = \mathcal{N}_0\, e^{-(i/\hbar)\,T_0}$. Moreover we find the equation

$$-s_{01}\sqrt{\Delta_{x1}^2\Delta_{p0}^2 - (\hbar/2)^2} = -s_0\sqrt{\Delta_{x0}^2\Delta_{p0}^2 - (\hbar/2)^2} + \Delta_{p0}^2\, T_0'' \tag{37}$$

which determines the new variance in x-direction (and also the sign s_{01}). Namely we have

$$\Delta_{x1}^2 = \Delta_{x0}^2 - 2s_0 T_0'' \sqrt{\Delta_{x0}^2 \Delta_{p0}^2 - (\hbar/2)^2} + \Delta_{p0}^2 [T_0'']^2 . \tag{38}$$

Repeating the same argumentation for the kick (the first term in Eq. (29)) we find

$$\begin{aligned} & e^{-(i/\hbar)\, V(x)}\, g(x_1, p_0, \Delta_{x1}, \Delta_{p0}, \mathcal{N}_{01}, s_{01}; x) \\ \approx\ & \mathcal{N}_{01} e^{-(i/\hbar)\, V_1}\, e^{-[A(\Delta_{x1}, \Delta_{p0}, s_{01}) + i V_1''/2\hbar](x-x_1)^2}\, e^{i[p_0 - V_1']\,(x-x_1)/\hbar} \\ =\ & \mathcal{N}_1\, g(x_1, p_1, \Delta_{x1}, \Delta_{p1}, \mathcal{N}_1, s_1; x) . \end{aligned} \tag{39}$$

This leads to $\mathcal{N}_1 = e^{-(i/\hbar)\, V_1} \mathcal{N}_{01}$, $p_1 = p_0 - V_1'$ and

$$\Delta_{p1}^2 = \Delta_{p0}^2 + 2s_{01} V_1'' \sqrt{\Delta_{x1}^2 \Delta_{p0}^2 - (\hbar/2)^2} + \Delta_{x1}^2 [V_1'']^2 . \tag{40}$$

Eqs. (38), (40) together with the classical equations of motion Eqs. (27), (28) and the transformation rules for the overall prefactor $\mathcal{N}$ and sign s represent a complete set of (classical) equations for the propagation of a wave packet in Gaussian approximation.

For the kicked rotator we have $T'' = 1$. Assuming a coherent initial state with $\Delta_{p0}^2 = \Delta_{x0}^2 = \hbar/2$ the equations for the transformed widths simplify to

$$\begin{aligned} \Delta_{x1}^2 &= \hbar \\ \Delta_{p1}^2 &= \left[\left(V_1'' - \frac{1}{2} \right)^2 + \frac{1}{4} \right] \hbar \end{aligned} \tag{41}$$

and we have $s_1 = \operatorname{sgn}\left(V_1'' - \frac{1}{2}\right)$. Finally we find for this special case

$$\begin{aligned} A(\Delta_{x1}, \Delta_{p1}, s_1) &= \frac{1 + \frac{2s_1 i}{\hbar} \sqrt{\left[\left(V_1'' - \frac{1}{2}\right)^2 + \frac{1}{4} \right] \hbar^2 - \left(\frac{\hbar}{2}\right)^2}}{4\hbar} \\ &= \frac{1 + 2i(V_1'' - \frac{1}{2})}{4\hbar} . \end{aligned} \tag{42}$$

With this result the calculation of the correlator Eq. (24) has been reduced to a simple Gaussian integration and we obtain the explicit expression

$$C_H(\xi_0; \xi_1) = \sqrt{\frac{8}{9 + [2V''(x_1) - 1]^2}} \tag{43}$$

which successfully compares to numerical data [4].

4 Conclusions

Quantum chaotic systems combine aspects of dynamical randomness with model-specific features that reflect the underlying deterministic classical dynamics. We studied this relation for the case of phase-space portraits (Husimi densities) of chaotic eigenstates and found that semiclassical considerations (Gaussian wave packet dynamics) require certain correlations which would not be expected from random-matrix theory. On the other hand, in order to estimate these correlations analytically, we had to employ assumptions from RMT. In other words, our strategy to describe a chaotic eigenstate is to use RMT whenever there is no "obvious" reason why it should fail. Clearly, this is only a pragmatic approach which lacks a solid justification but is unavoidable as long as there is no complete semiclassical theory for chaotic eigenstates (or, equivalently, for the quantum dynamics beyond the Heisenberg time). For the kicked rotator the results of this "hybrid" method have been confirmed numerically in [4] and we expect similar agreement also in other models such as quantum billiards. Moreover, an analogous strategy had been employed earlier by Kaplan and Heller in order to account for scarring on short periodic orbits [6].

The phase-space correlations which we are describing in the present contribution are semiclassically strong and long range in the sense that (i) the correlator does not decay to zero in the semiclassical limit and that (ii) the correlated points can have an arbitrarily large separtion in phase space as long as they are connected by a classical trajectory which is shorter than the time t^* for quantum classical-correspondence. This is an important difference to eigenstate correlations in other representations, in particular to spatial correlations which are experimentally easier accessible and thus received a lot of attention in the past. Some potential applications for phase-space correlations have been pointed out in [4].

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